basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2024

MARKS: 150
PUNTE: 150

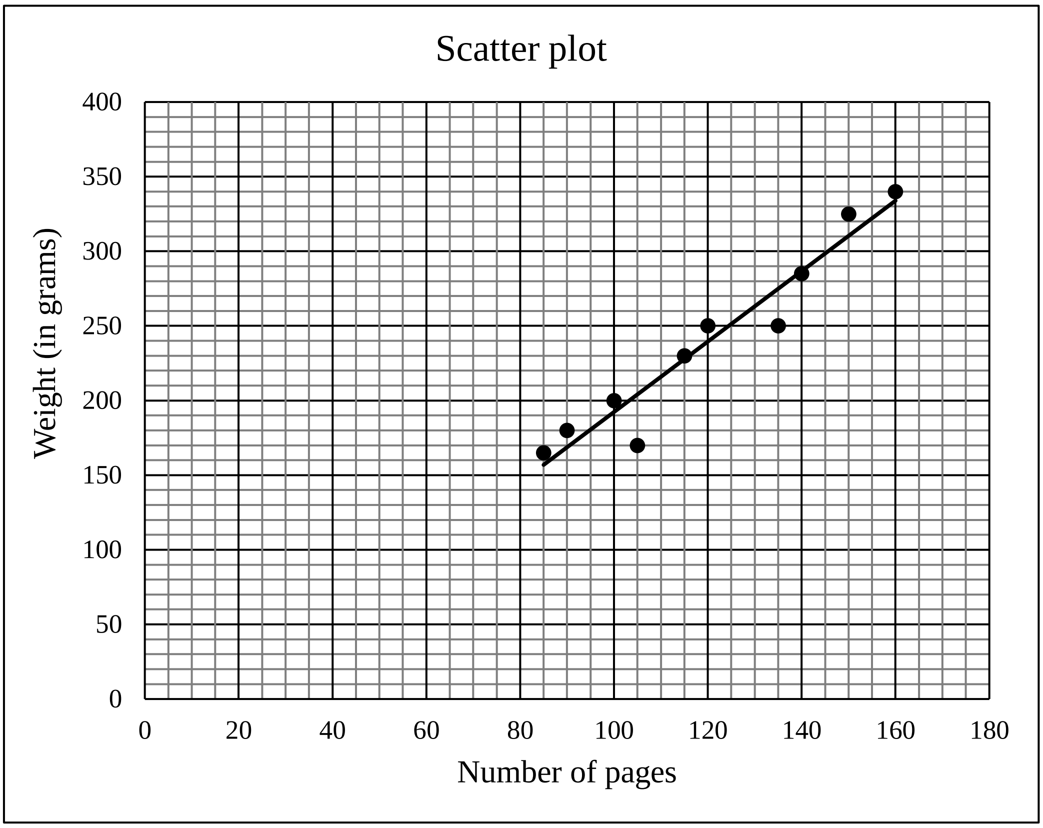
These marking guidelines consist of 26 pages./
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and did not redo the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord op 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a=-43,72$ $b=2,36$ $y=-43,72+2,36x$	✓ $a=-43,72$ ✓ $b=2,36$ ✓ equation (3)
1.2		✓ any correct two points ✓ straight line joining the points for $x\in[85\ ;160]$ (2)
1.3	$y=-43,72+2,36(110)$ $y=215,88$ **OR** y = 215,90 (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$y=-43,72+2,36(130)$ $y=263,08$ Percentage increase in weight $=\dfrac{263,08-215,88}{215,88}\times 100$ $=21,86\%$ **OR** $y=263,08$ Percentage $=\dfrac{263,08}{215,88}\times 100$ $=121,86\ \%$ Percentage increase in weight $=121,86-100=21,86$	✓ y-value ✓ difference between y-values ✓ +ve answer (3) ✓ y-value ✓ difference between % ✓ +ve answer (3)
		[10]

QUESTION/*VRAAG* 2

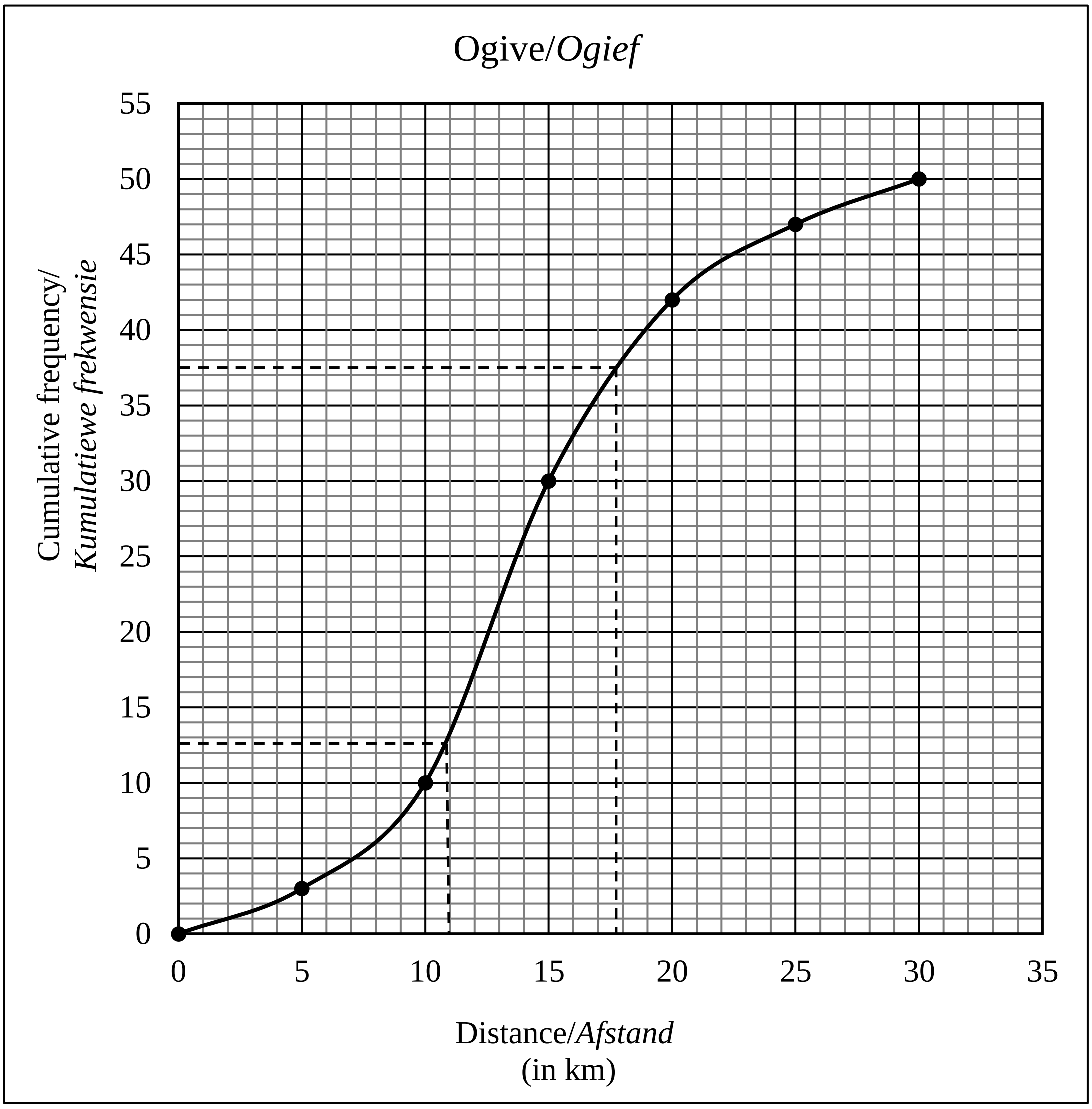
2.1

Distance (x km)	Frequency	Cumulative frequency
$0 \le x < 5$	3	3
$5 \le x < 10$	7	10
$10 \le x < 15$	20	30
$15 \le x < 20$	12	42
$20 \le x < 25$	5	47
$25 \le x < 30$	3	50

✓ 10
✓ all values correct

(2)

2.2



✓ grounding

✓ plotting a min of 3 points (cf at upper limits)

✓ smooth, increasing curve

(3)

2.3

$Q_3 = 17{,}8$

$Q_1 = 11$

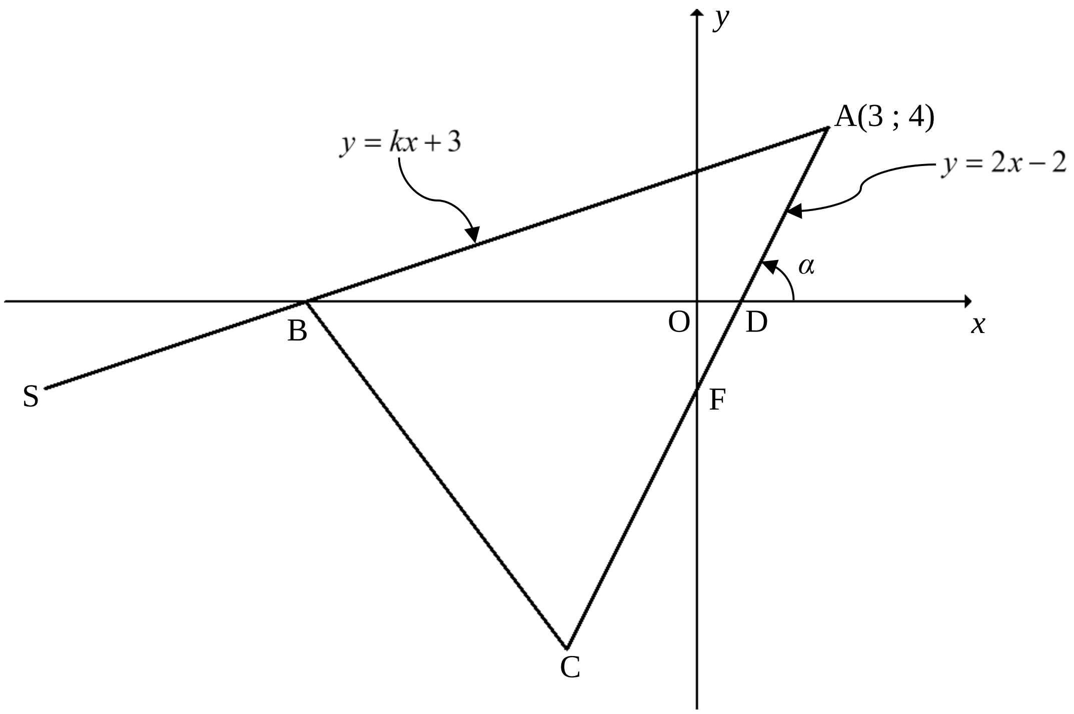
IQR = 6,8

✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5)
✓ answer (accept 5-9)

(2)

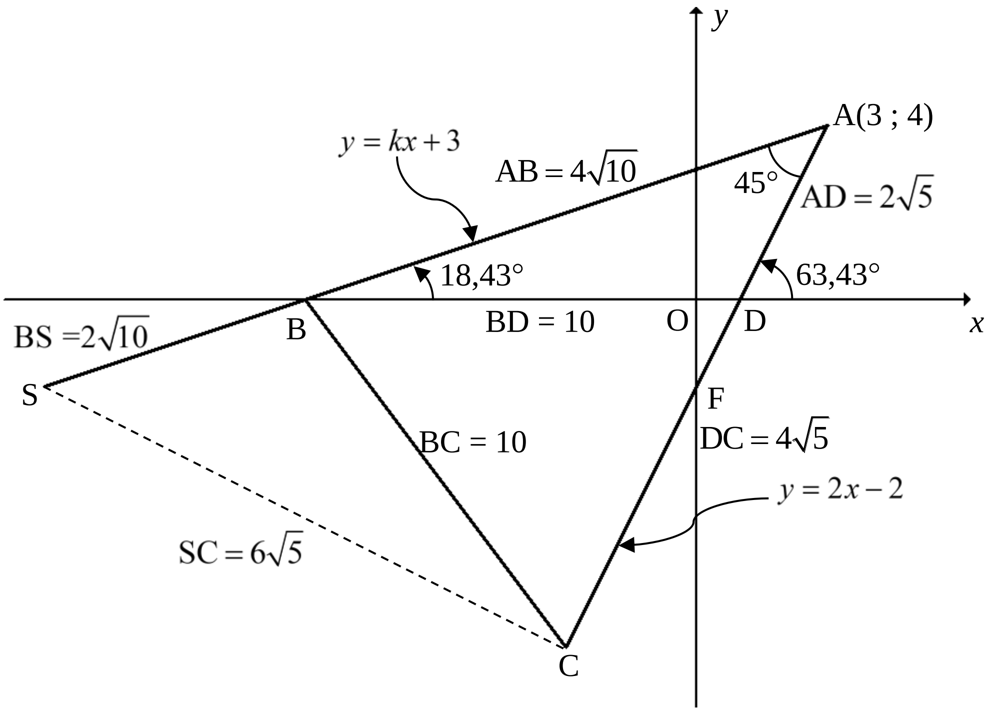
2.4	$5 \leq x < 10$	✓ $5 \leq x < 10$ (1)
2.5	Estimated mean $= \dfrac{2,5(3)+7,5(11)+12,5(20)+17,5(8)+22,5(5)+27,5(3)}{50}$ $= \dfrac{675}{50}$ $= 13,5$ km	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

QUESTION/*VRAAG* 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ **OR** y-intercept of AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) (1) ✓ substitution (0 ; 3) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ $B(-9\ ;\ 0)$	✓ $y = 0$ ✓ answer (2)

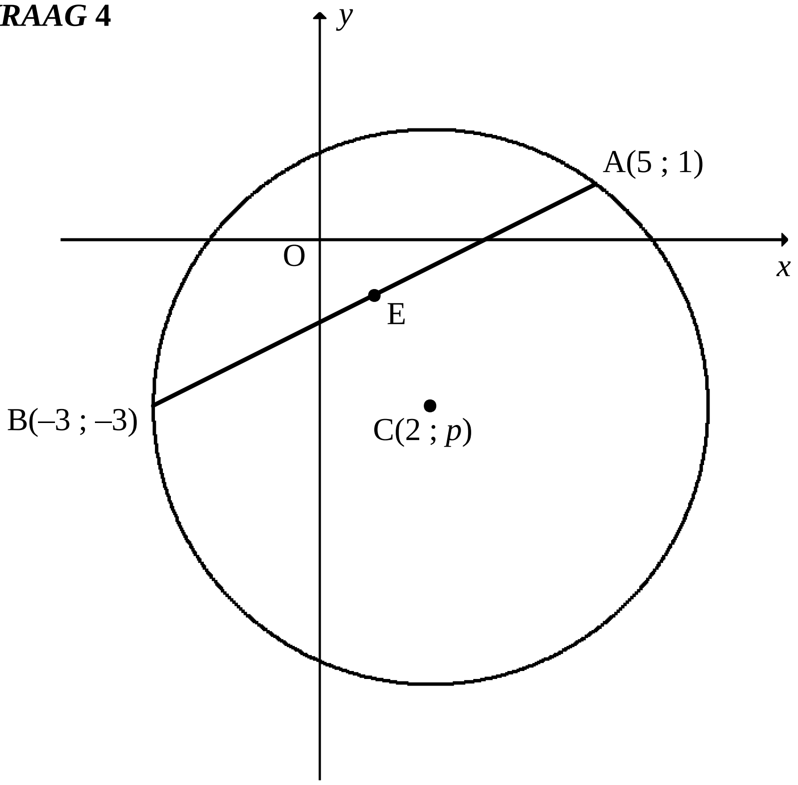
3.3	$F(0\,;-2)$ $F\left(\frac{x+3}{2}\ ;\ \frac{y+4}{2}\right)$ $\frac{x+3}{2}=0 \qquad \frac{y+4}{2}=-2$ $x=-3 \qquad y=-8$ $C(-3\,;-8)$ **OR** by translation $F(0;-2)$ $A \to F\ (x;y) \to (x-3;y-6)$ $F \to C(0;-2) \to (0-3;-2-6)=(-3;-8)$	✓ $F(0\,;-2)$ ✓ $\frac{x+3}{2}=0\ ;\ \frac{y+4}{2}=-2$ ✓ x-value ✓ y-value (4) ✓ $F(0;-2)$ ✓ $(x-3\ ;y-6)$ ✓ x-value ✓ y-value (4)
3.4	$m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y=-\frac{4}{3}x+c$ $(-2)=-\frac{4}{3}(-15)+c$ $c=-22$ $y=-\frac{4}{3}x-22$ **OR** $m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y-y_1=-\frac{4}{3}(x-x_1)$ $y-(-2)=-\frac{4}{3}(x-(-15))$ $y+2=-\frac{4}{3}x-20$ $y=-\frac{4}{3}x-22$	✓ substitution of B and C into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5) ✓ substitution into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5)



3.5	$\tan\alpha = m_{AC} = 2$ $\alpha = 63,43°$ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ $A\hat{B}D = 18,43°$ $B\hat{A}C = \alpha - A\hat{B}D$ $B\hat{A}C = 63,43° - 18,43°$ $B\hat{A}C = 45°$	✓ $\tan\alpha = m_{AC} = 2$ ✓ $\alpha = 63,43°$ ✓ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ ✓ $A\hat{B}D = 18,43°$ ✓ answer (5)
	OR $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ BD = 10 $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB.AD\cos B\hat{A}C$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5})\cos B\hat{A}C$ $\cos B\hat{A}C = \frac{\sqrt{2}}{2}$ $B\hat{A}C = 45°$	✓ length of AB ✓ calculation of remaining 2 lengths ✓ substitution into cosine-rule ✓ rewriting in terms of $\cos B\hat{A}C$ ✓ answer (5)

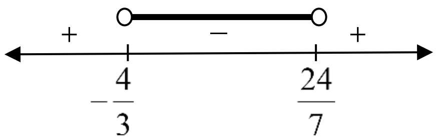
3.6	A(3 ; 4) and S(–15 ; – 2) $\text{AS} = \sqrt{(x_A - x_S)^2 + (y_A - y_S)^2}$ $\text{AS} = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ $\text{AS} = \sqrt{360} = 6\sqrt{10} = 18,97$ $\dfrac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{ASC}} = \dfrac{\frac{1}{2}(\text{BD})(\perp \text{h})}{\frac{1}{2}(\text{AS})(\text{AC})\sin \text{B}\hat{\text{A}}\text{C}}$ $\dfrac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{ASC}} = \dfrac{\frac{1}{2}(10)(4)}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin 45°}$ $\dfrac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{ASC}} = \dfrac{2}{9}$	✓ $\text{AS} = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
	OR $\text{AS} = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ $\text{AS} = \sqrt{360} = 6\sqrt{10} = 18,97$ $\text{AB} = \sqrt{(-9-3)^2 + (0-4)^2} = 4\sqrt{10}$ $\text{AD} = \sqrt{(3-1)^2 + (4-0)^2} = 2\sqrt{5}$ $\dfrac{\text{Area of } \Delta\text{ABD}}{\text{Area of } \Delta\text{ASC}} = \dfrac{\frac{1}{2}(\text{AB})(\text{AD})\sin\hat{\text{A}}}{\frac{1}{2}(\text{AS})(\text{AC})\sin\hat{\text{A}}}$ $= \dfrac{\frac{1}{2}(4\sqrt{10})(2\sqrt{5})\sin\hat{\text{A}}}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin\hat{\text{A}}}$ $= \dfrac{2}{9}$	✓ $\text{AS} = \sqrt{(3-(-15))^2 + (4-(-2))^2}$ ✓ length of AS ✓ Area ΔABD ✓ Area ΔASC ✓ answer (5)
		[22]

QUESTION/*VRAAG* 4



4.1	$E\left(\frac{5+(-3)}{2};\frac{1+(-3)}{2}\right)$ $\therefore E(1;-1)$	✓ $x=1$ ✓ $y=-1$ (2)
4.2	$AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$ $AB=\sqrt{(5-(-3))^2+(1-(-3))^2}$ $AB=\sqrt{80}=4\sqrt{5}=8,94$ units	✓ $AB=\sqrt{80}=4\sqrt{5}=8,94$ (1)
4.3	$m_{AB}=\frac{1-(-3)}{5-(-3)}$ $m_{AB}=\frac{1}{2}$ $\therefore m_{CE}=-2$ [$CE\perp AB$] $E(1;-1)$ $y=-2x+c$ **OR** $y-y_1=-2(x-x_1)$ $(-1)=-2(1)+c$ $y-(-1)=-2(x-1)$ $c=1$ $y=-2x+1$ $y=-2x+1$	✓ $m_{AB}=\frac{1}{2}$ ✓ m_{CE} ✓ substitution of E ✓ equation (4)

4.4	$y=-2x+1$ $p=-2(2)+1$ $p=-3$ **OR** $m_{\text{CE}}=-2$ $\frac{p-(-1)}{2-1}=-2$ $p+1=-2$ $p=-3$	✓ substitution of C(2 ; p) into $\perp$ bisector of AB (1) ✓ substitution of C and E into the gradient formula (1)
4.5	BC = r = 5 units $\therefore (x-2)^2+(y+3)^2=25$ $x^2-4x+4+y^2+6y+9=25$ $x^2+y^2-4x+6y-12=0$	✓ BC = r = 5 units ✓ $(x-2)^2+(y+3)^2$ ✓ r^2 ✓ $x^2-4x+4+y^2+6y+9=25$ (4)

4.6	$(x-2)^2+(y+3)^2=25$ **OR** $x^2+y^2-4x+6y-12=0$ $y=tx+8$ $(x-2)^2+(tx+8+3)^2=25$ **OR** $x^2+(tx+8)^2-4x+6(tx+8)-12=0$ $x^2-4x+4+t^2x^2+22tx+121-25=0$ **OR** $x^2+t^2x^2+16tx+64-4x+6tx+48-12=0$ $x^2(t^2+1)+x(22t-4)+100=0$ $\Delta<0$ $(22t-4)^2-4(t^2+1)(100)<0$ $484t^2-176t+16-400t^2-400<0$ $84t^2-176t-384<0$ $21t^2-44t-96<0$ $(7t-24)(3t+4)<0$ CV: $\frac{24}{7}$; $-\frac{4}{3}$  $\therefore t\in\left(-\frac{4}{3}\ ;\ \frac{24}{7}\right)$ **OR** $-\frac{4}{3}<t<\frac{24}{7}$	✓ substitution of $y=tx+8$ ✓ standard form ✓ $\Delta<0$ ✓ standard form of Δ ✓ critical values ✓ answer (6)
		[18]

QUESTION/*VRAAG* 5

5.1.1	$\sin 220°$ $= -\sin 40°$ $= -p$	✓ $-\sin 40°$ ✓ answer (2)
5.1.2	$\cos^2 50°$ $= \sin^2 40°$ $= p^2$	✓ $\sin^2 40$ ✓ answer (2)
5.1.3	$\cos(-80°)$ $= \cos 80°$ $= 1 - 2\sin^2 40°$ $= 1 - 2p^2$ **OR** $\cos(-80°)$ $= \cos 80°$ $= \cos(30° + 50°)$ $= \cos 30° \cos 50° - \sin 30° \sin 50°$ $= \frac{\sqrt{3}p}{2} - \frac{\sqrt{1-p^2}}{2}$	✓ $\cos 80°$ ✓ double angle ✓ answer (3) ✓ $\cos 80°$ ✓ expansion ✓ answer (3)
5.2.1	LHS = $\tan x(1 - \cos^2 x) + \cos^2 x$ $= \frac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$ $= \frac{\sin^3 x + \cos^3 x}{\cos x}$ $= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cos x}$ $= \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$ $=$ RHS **OR**	✓ $\frac{\sin x}{\cos x}$ ✓ $\sin^2 x$ ✓ simplification ✓ factorisation of cubes ✓ $\sin^2 x + \cos^2 x = 1$ (5)

	$\text{RHS}=\frac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$ $=\frac{\sin x-\sin^2 x\cos x+\cos x-\sin x\cos^2 x}{\cos x}$ $=\tan x-\sin^2 x+1-\sin x\cos x$ $=\tan x+\cos^2 x-\sin x\cos x$ $=\tan x\left(1-\frac{\sin x\cos x}{\tan x}\right)+\cos^2 x$ $=\tan x\left(1-\frac{\sin x\cos x}{\frac{\sin x}{\cos x}}\right)+\cos^2 x$ $=\tan x\left(1-\cos^2 x\right)+\cos^2 x$ $=\text{LHS}$	✓ multiplication ✓ $\div$ by $\cos x$ ✓ $-\sin^2 x+1=\cos^2 x$ ✓ factorisation ✓ $\tan x=\frac{\sin x}{\cos x}$ (5)
5.2.2	$\cos x=0$ or where $\tan x$ is undefined $x=90°+k.360°$ or $x=270°+k.360°$ $x=90°$ or $x=-90°$	✓ $\cos x=0$ or $\tan x$ undefined ✓ $x=90°$ ✓ $x=-90°$ (3)
5.3.1	$\frac{\sin 150°+\cos^2 x-1}{2}$ $=\frac{\sin 30°+\cos^2 x-1}{2}$ $=\frac{\frac{1}{2}-\left(1-\cos^2 x\right)}{2}$ $=\left(\frac{1}{2}-\sin^2 x\right)\times\frac{1}{2}$ $=\frac{1-2\sin^2 x}{4}$ $=\frac{\cos 2x}{4}$	✓ $\sin 30°$ ✓ $\sin 30°=\frac{1}{2}$ ✓ factor ✓ $1-\cos^2 x=\sin^2 x$ ✓ simplification ✓ answer in terms of cos $2x$ (6)
5.3.2	$\frac{\sin 150°+\cos^2 x-1}{2}=\frac{1}{25}$ $\frac{\cos 2x}{4}=\frac{1}{25}$ $\cos 2x=\frac{4}{25}$ $\text{ref}\angle=80,79...°$ $2x=80,79...°+k.360°$ or $2x=279,20...°+k.360°$ $x=40,40°+k.180°$ or $x=139,60°+k.180°$; $k\in\mathbb{Z}$	✓ answer $5.3.1=\frac{1}{25}$ ✓ $2x=80,79°$ ✓ $2x=279,20...°$ ✓ $x=40,40°$ and $x=139,60°$ ✓ $+k.180°$; $k\in\mathbb{Z}$ (5)

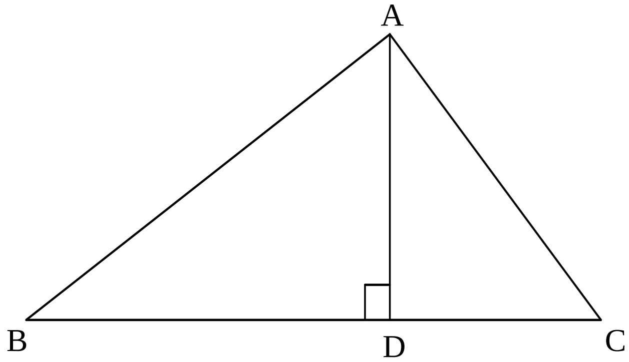
	OR $\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$ $\sin 150° + \cos^2 x - 1 = \frac{2}{25}$ $\sin 30° + \cos^2 x - 1 = \frac{2}{25}$ $\cos^2 x = \frac{29}{50}$ $\cos x = \pm\sqrt{\frac{29}{50}}$ $x = 40,40° + k.360°$ or $x = 319,60° + k.360°$; $k \in Z$ or $x = 139,60° + k.360°$ or $x = 220,40° + k.360°$; $k \in Z$	✓ $\cos^2 x = \frac{29}{50}$ ✓ $x = 40,40°$ ✓ $x = 139,60°$ ✓ $x = 220,40°$ and $x = 319,60°$ ✓ $+ k.360°$; $k \in Z$ (5)
		[26]

QUESTION/*VRAAG* 6

6.1	Period = $360°$	✓ $360°$ (1)
6.2	Amplitude $= 1$	✓ 1 (1)
6.3	$a = -45°$	✓ $a = -45°$ (1)
6.4	$\sin 2x = k$ $k = \sin(2\times 165°)$ **OR** $k = \sin\left(2\times(-75°)\right)$ $k = \sin 330°$ $\quad$ $k = \sin(-150°)$ $k = -\sin 30°$ $k = -\frac{1}{2}$ **OR** $k = \cos(165° - 45°)$ **OR** $k = \cos(-75° - 45°)$ $k = \cos 120°$ $\quad$ $k = \cos(-120°)$ $k = -\cos 60°$ $k = -\frac{1}{2}$	✓ $-\sin 30°$ ✓ $-\frac{1}{2}$ (2) ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ (2)
6.5	Points of intersection are translated $60°$ to the left $x = -15°$	✓ $x = -15°$ (1)
6.6	$\sqrt{2}\sin 2x = \sin x + \cos x$ $\sin 2x = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$ $\sin 2x = \sin 45° \sin x + \cos 45° \cos x$ $\sin 2x = \cos(45° - x)$ **OR** $\sin 2x = \cos(x - 45°)$ $\therefore$ 2 roots in the interval $x \in [-90°; 90°]$	✓ division by $\sqrt{2}$ ✓ special angles ✓ $\cos(45° - x)$ or $\cos(x - 45°)$ ✓ answer (4)
		[10]

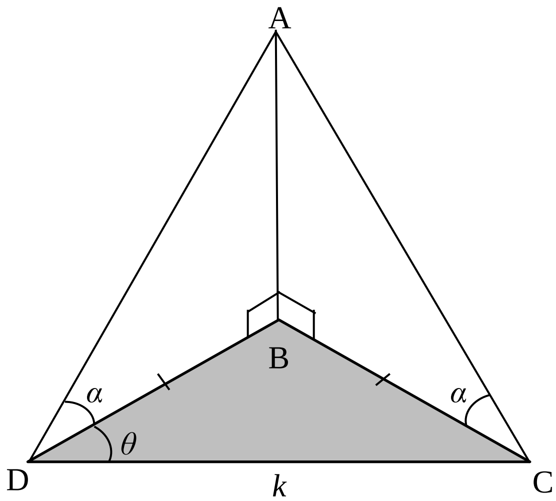
QUESTION/*VRAAG* 7

7.1



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB \sin \hat{B}$	✓ $\sin \hat{B} = \frac{AD}{AB}$ ✓ answer (2)
7.1.2	Area of $\Delta ABC = \frac{1}{2}(BC)(AD)$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$	✓ $\frac{1}{2}(BC)(AD)$ (1)

7.2



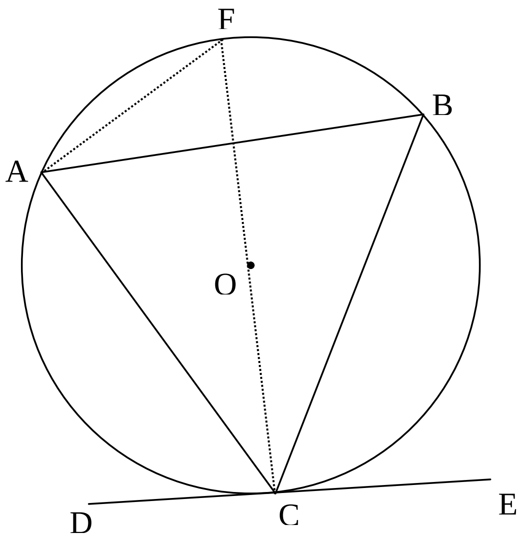
7.2.1	In ΔADB $\sin \alpha = \frac{AB}{AD}$ $AD = \frac{AB}{\sin \alpha}$ In ΔABC $\sin \alpha = \frac{AB}{AC}$ $AC = \frac{AB}{\sin \alpha}$ $AD = AC$ **OR** In ΔADB and ΔACB	✓ $\sin \alpha = \frac{AB}{AD}$ ✓ $\sin \alpha = \frac{AB}{AC}$ (2)

	AB = AB [common side] $\hat{ABD} = \hat{ABC} = 90°$ [given] BD = BC [given] $\Delta ADB \equiv \Delta ACB$ [S∠S] $\therefore$ AD = AC	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR In ΔADB and ΔACB $A\hat{D}B = A\hat{C}B = \alpha$ [given] $A\hat{B}D = A\hat{B}C = 90°$ [given] AB = AB **OR** BD = BC [common side **OR** given] $\therefore$ $\Delta ADB \equiv \Delta ACB$ [∠∠S] $\therefore$ AD = AC	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR $AD^2 = AB^2 + DB^2$ [Pythagoras] $AC^2 = AB^2 + BC^2$ [Pythagoras] But DB = BC [given] $\therefore AD^2 = AC^2$ $\therefore$ AD = AC	✓ both Pythagoras statements ✓ DB = BC (2)
7.2.2	$\frac{BD}{\sin\theta} = \frac{k}{\sin(180° - 2\theta)}$ $BD = \frac{k\sin\theta}{\sin 2\theta}$ $BD = \frac{k\sin\theta}{2\sin\theta\cos\theta}$ $BD = \frac{k}{2\cos\theta}$	✓ substitution of $(180° - 2\theta)$ into sine rule ✓ reduction ✓ double angle (3)
	OR $BC^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $BD^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $k^2 - 2k(BD)\cos\theta = 0$ $2k(BD)\cos\theta = k^2$ $\therefore BD = \frac{k}{2\cos\theta}$	✓ substitution into cosine-rule ✓ substitution BC with BD into cosine-rule ✓ simplification in terms of BD (3)

7.2.3	Area of $\Delta\text{BCD}=\frac{1}{2}(\text{DC})(\text{BD})\left(\sin \text{C}\hat{\text{D}}\text{B}\right)$ $=\frac{1}{2}k\left(\frac{k}{2\cos\theta}\right)\sin\theta$ $=\frac{1}{4}k^2\tan\theta$ **OR** Area of $\Delta\text{BCD} = \frac{1}{2}(\text{BD})(\text{BC})\left(\sin\left(180°-2\theta\right)\right)$ $= \frac{1}{2}\left(\frac{k}{2\cos\theta}\right)\left(\frac{k}{2\cos\theta}\right)(\sin 2\theta)$ $= \frac{2k^2\sin\theta\cos\theta}{8\cos\theta\cos\theta}$ $= \frac{1}{4}k^2\tan\theta$	✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta}=\tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3) ✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta}=\tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3)
		[11]

QUESTION/*VRAAG* 8

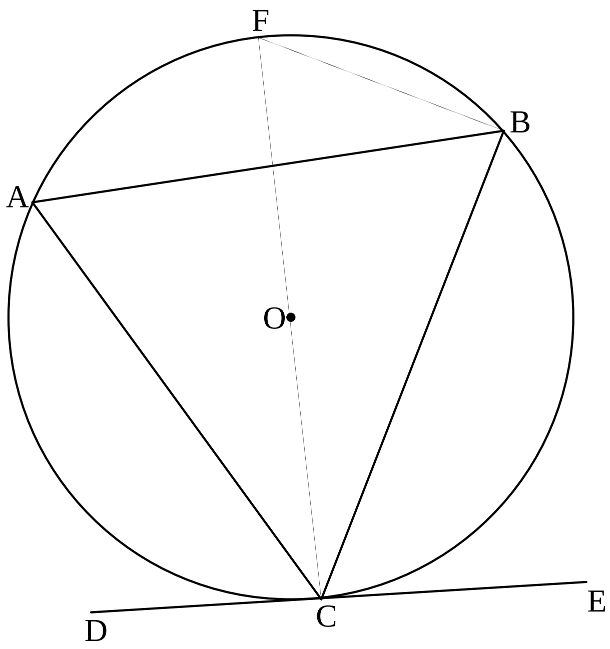
8.1



	Construction: Draw diameter CF and draw AF *Konstruksie: Trek middellyn* CF *en verbind* AF $F\hat{C}E = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*] $F\hat{A}C = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $F\hat{A}B = F\hat{C}B$ [∠s same segment/∠*e dieselfde segm*] $\therefore B\hat{A}C = B\hat{C}E$ $\therefore B\hat{C}E = \hat{A}$	✓Constr ✓ S ✓ R ✓ S/R ✓ S/R (5)

OR

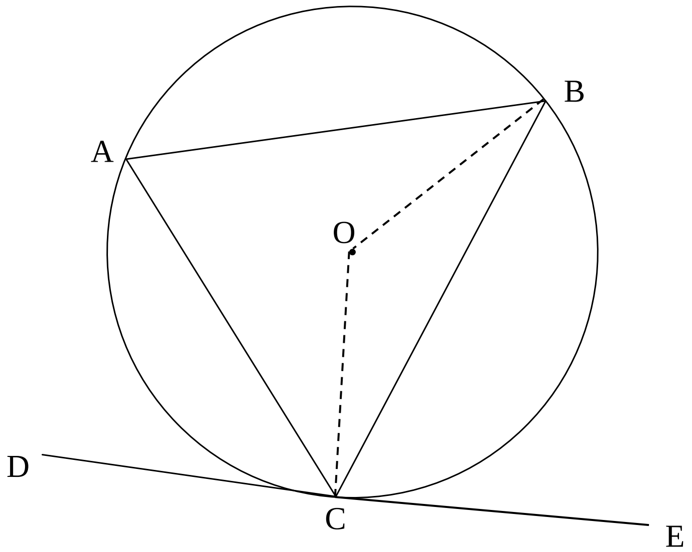
8.1



	Construction: Draw diameter CF and draw FB *Konstruksie: Trek middellyn* CF *en verbind* FB $F\hat{B}C = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $B\hat{F}C + F\hat{C}B = 90°$ [sum of ∠s in Δ/*binne ∠e v Δ*] $O\hat{C}E = 90°$ [tan ⊥ radius/ *raaklyn ⊥ radius*] $\therefore B\hat{C}E = \hat{F}$ but $\hat{A} = \hat{F}$ [∠s in same seg/∠ *in dies. segment*] $\therefore B\hat{C}E = \hat{A}$	✓ construction ✓ S / R ✓ S ✓ R ✓ S / R (5)

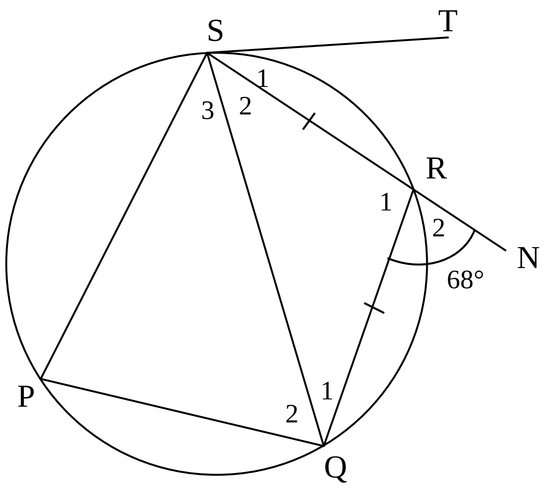
OR

8.1



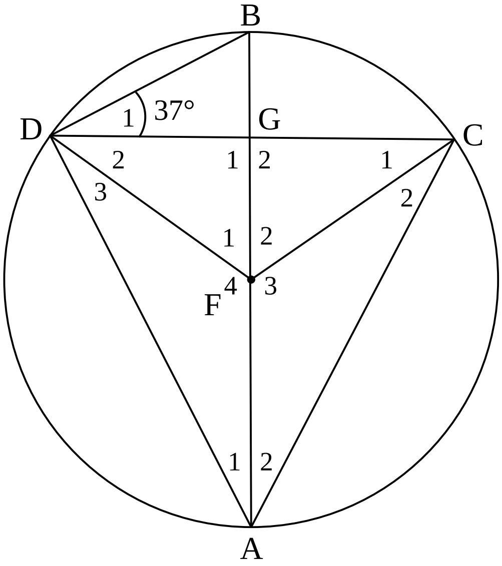
	Construction: Draw radii BO and OC *Konstruksie: Trek radiusse BO en OC* $O\hat{C}E = 90°$ or $B\hat{C}E = 90° - O\hat{C}B$ [tan ⊥ radius / *raaklyn ⊥ radius*] $O\hat{C}B = O\hat{B}C$ [∠s opp equal sides/ *∠e teenoor gelyke sye*] $\therefore\ C\hat{O}B = 180° - 2O\hat{C}B$ [∠s of Δ/*∠e van Δ*] $C\hat{A}B = 90° - O\hat{C}B$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore B\hat{C}E = C\hat{A}B$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

8.2



8.2.1	$\hat{P} = \hat{R}_2 = 68°$ [ext ∠ of cyclic quad /*buite ∠ van kvh*]	✓ S ✓ R (2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $\hat{Q}_1 + \hat{S}_2 = 68°$ [ext ∠ of Δ/ *buite ∠ van* Δ] $\therefore \hat{Q}_1 = 34°$	✓ S ✓ S (2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$ [tan-chord theorem/∠ *tussen rkl en koord*]	✓ S ✓ R (2)
		[11]

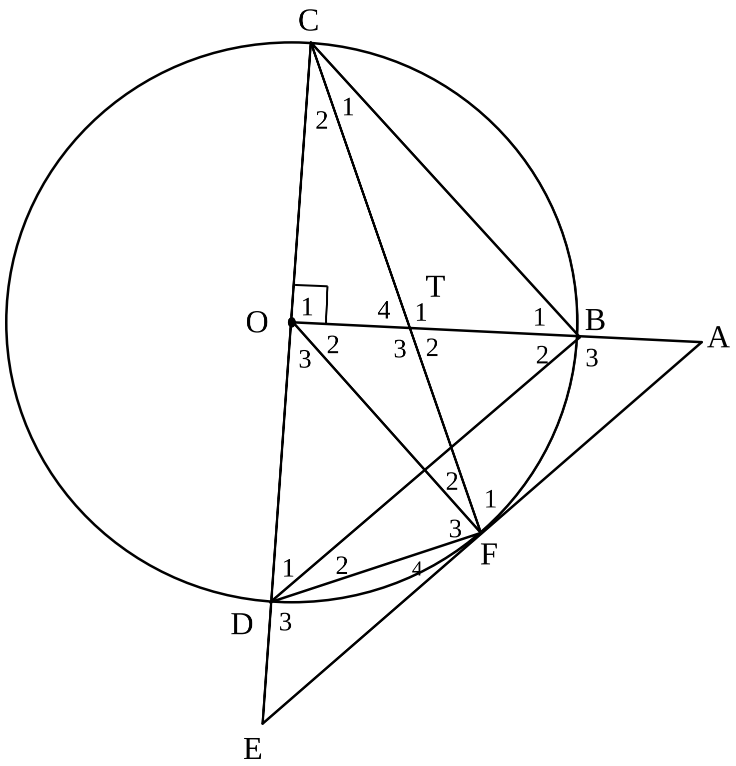
QUESTION/*VRAAG* 9



9.1	$\hat{A}_2 = \hat{D}_1 = 37°$ [∠s in the same seg/∠*e in dies segment*] $\hat{A}_1 = \hat{A}_2 = 37°$ [BA bisects $C\hat{A}D$/BA *halveer* $C\hat{A}D$] $\hat{D}_3 = \hat{A}_1 = 37°$ [∠s opp equal sides/∠*e teenoor gelyke sye*] $\hat{C}_2 = \hat{A}_2 = 37°$ [∠s opp equal sides/∠*e teenoor gelyke sye*]	✓ S ✓ R ✓✓ any other two statements (4)
9.2	$A\hat{D}G = 53°$ [∠ in semi circle/∠ *in halwe sirkel*] $\hat{A}_1 = 37°$ [proved in 9.1/*reeds bewys in 9.1*] $\therefore \hat{G}_1 = 90°$ [sum of ∠s in Δ/*binne* ∠*e van* Δ] $\therefore$ CG = DG [line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ *op koord*] **OR** $\hat{F}_2 = 2\hat{D}_1 = 74°$ [∠ at centre = 2 ×∠ at circumference/ *midpt.* ∠*s = 2×omtreks*∠] $\hat{D}_3 = 37°$ [proved in 9.1/*reeds bewys in 9.1*] $\therefore \hat{D}_2 = 16°$ [∠ in semi circle/∠ *in halwe sirkel*] $\hat{C}_1 = \hat{D}_2 = 16°$ [∠s opp equal sides/∠*e teenoor gelyke sye*] $\therefore \hat{G}_2 = 90°$ [sum of ∠s in Δ/*binne* ∠*e van* Δ] $\therefore$ CG = DG [line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ op *koord*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$ [∠ at centre = 2 ×∠ at circum./ *midpt. ∠s = 2×omtreks∠*] $\frac{FG}{20} = \cos 74°$ $FG = 5,51$ $\therefore BG = 14,49$ units **OR** $\hat{F}_2 = 2\hat{D}_1 = 74°$ [∠ at centre = 2 ×∠ at circumference *midpt. ∠= 2×omtreks∠*] $\frac{FG}{20} = \sin 16°$ $FG = 5,51$ $\therefore BG = 14,49$ units **OR** $\frac{DG}{20} = \cos 16°$ DG = 19,23 $\frac{BG}{19,23} = \tan 37°$ BG = 14,49 units **OR** $\frac{DG}{20} = \cos 16°$ DG = 19,23 $FG^2 = FD^2 - DG^2$ [Pythagoras] $FG^2 = 20^2 - (19,23)^2$ FG = 5,51 BG = $20 - 5,51$ = 14,49 units	✓ S ✓ trig ratio ✓ FG ✓ answer (4) ✓ S ✓ trig ratio ✓ FG ✓ answer (4) ✓ trig ratio ✓ length of DG ✓ trig ratio ✓ answer (4) ✓ trig ratio ✓ length of DG ✓ correct use of Pythagoras ✓ answer (4)
		[12]

QUESTION/*VRAAG* 10



10.1	$\hat{O}_1 = 90°$ [given/*gegee*] $\hat{F}_2 + \hat{F}_3 = 90°$ [$\angle$ in semi circle/$\angle$ *in halwe sirkel*] $\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$ $\therefore$ TODF is a cyclic quad [ext $\angle$ = int opp $\angle$/ *buite* $\angle$ = *teenoorst. binne* $\angle$] **OR** [converse ext $\angle$ of cyclic quad/ *omgekeerde buite* $\angle$ *v kvh*]	 ✓ S ✓ R ✓ S ✓ R (4)
10.2	$\hat{T}_1 = \hat{T}_3$ [vert opp $\angle$s =/ *regoorstaande* $\angle$*e*] But $\hat{D}_3 = \hat{T}_3$ [ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*] $\therefore \hat{T}_1 = \hat{D}_3$	✓ S / R ✓ S ✓ R (3)
10.3	In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ [ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*] 2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/ $\angle$ *tussen rkl en koord*] but $\hat{C}_2 = \hat{F}_2$ [$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*] $\therefore \hat{F}_4 = \hat{F}_2$ 3) $\hat{E} = \hat{O}_2$ [3rd $\angle$ of Δ/$\angle$*e van* Δ] ΔTFO ||| ΔDFE [$\angle\angle\angle$]	 ✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)

	OR In ΔDFE and ΔTFO 1) $\hat{D}_3 = \hat{T}_3$ [ext ∠ of cyclic quad/*buite ∠ van Δ*] 2) $\hat{F}_4 = \hat{C}_2$ [tan-chord theorem/∠ *tussen rkl en koord*] $\hat{F}_2 + \hat{F}_3 = 90°$ [∠ in semi circle/∠ *in halwe sirkel*] $\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$ [sum of ∠s in Δ/ *binne ∠e van Δ*] $\hat{E} = 90° - 2\hat{F}_4$ [ext ∠ of Δ/ *buite ∠ van Δ*] $\hat{O}_3 = 2\hat{C}_2$ [∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*] $\hat{O}_2 = 90° - 2\hat{F}_4$ [∠s on a str line/∠*e op 'n reguitlyn*] $\hat{O}_2 = \hat{E}$ 3) $\therefore \hat{F}_4 = \hat{F}_2$ [3rd ∠ of Δ/∠*e van Δ*] ΔTFO ||| ΔDFE [∠∠∠]	✓ S ✓ S / R ✓ S ✓ S ✓ S **OR** R (5)
10.4	$\hat{B}_2 = \hat{D}_1$ [∠s opp equal sides/∠*e teenoor gelyke sye*] $\hat{B}_2 = \hat{E}$ [given/*gegee*] $\therefore \hat{D}_1 = \hat{E}$ ∴DB || EA [corresp ∠s =/*ooreenkomstige ∠e gelyk*]	✓ S / R ✓ R (2)
10.5	In ΔOEA DB || EA [proven/*reeds bewys*] $\frac{OD}{DE} = \frac{OB}{BA}$ [line || one side of Δ/*lyn || een sy van Δ*] **OR** [prop theorem; DB || EA/ *eweredigheid stelling; DB || EA*] $\therefore DE = \frac{DO.AB}{OB}$ $\frac{FO}{FE} = \frac{TO}{DE}$ [ΔTFO ||| ΔDFE] $DE = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$ $\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$ [DO = OB = FO] $\therefore DO = \frac{TO.FE}{AB}$	✓ R ✓ S ✓ S / R ✓ S ✓ S (5)
		[19]

TOTAL/*TOTAAL*: 150